Computational investigation of conjugate heat transfer in cavity filled with saturated porous media

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The conjugate natural convection heat transfer in a partially heated porous enclosure had been studied numerically. The governing dimensionless equations are solved using finite element method. Classical Darcy model has been used and the considering dimensionless parameters are modified Rayleigh number ($10 \leq Ra \leq 10^3$), finite wall thickness ($0.02 \leq D \leq 0.5$), thermal conductivity ratio ($0.1 \leq K_r \leq 10$), and the aspect ratio ($0.5 \leq A \leq 10$). The results are presented in terms of streamlines, isotherms and local and average Nusselt number. The results indicate that heat transfer can be enhanced by increasing the modified Rayleigh number, and thermal conductivity ratio. Wall thickness effects on the heat transfer mechanism had been studied and it is found that; as the Wall thickness increases, the conduction heat transfer mechanism will be dominated. Also, increasing aspect ratio will increase the stream function and reduced the heat transfer rate.

**Keywords:** partially heated, conjugate, porous cavity, aspect ratio, finite element method.
investigations focusing on the convection heat transfer in porous media using nanofluid as working fluid (Sheikholeslami and Seyednezhad 2018; Sheikholeslami and Rokni 2018; Sheikholeslami 2018; Sheikholeslami and Shehzad 2018; Al-Farhany and Abdulkadhim 2018).

It can be noticed from the literature review and according to the best author's knowledge that there were limitations in the studies regarding partially active walls of enclosures so this is the motivation for the present work. In this way, the main objective of the present work is to describe the natural convection heat transfer in porous enclosure partially heated from the left side wall and how the finite thickness wall effect on heat transfer rate. The finite element method used to study the effect of various dimensionless parameters such as modified Rayleigh number, thermal conductivity ratio, thickness wall and the aspect ratio on the natural convection heat transfer characteristics. The results are presented in terms of streamline, isotherms, local and average Nusselt number.

2. MATHEMATICAL and COMPUTATIONAL MODEL

2.1 MATHEMATICAL FORMULATION

In this paper, two-dimension natural convection on porous cavity have been studded numerically with the effect of the partially heated conduction on vertical wall as shown in Fig. 1

The governing equations are subjected according to the assumptions:

1. The flow is considered to be two-dimensional fluid flow, laminar, and steady state.
2. The enclosure walls are impermeable.
3. The porous media is homogenous and isotropic.
4. The local thermal equilibrium is applied for porous matrix and the fluid.
5. Darcy model is applied for predictions of fluid flow inside the porous medium.
6. The internal heat generation assumed to be neglected.

The Continuity, momentum and energy of two-dimensional steady state natural convection in porous cavity equations are:

The Continuity equation is:
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  

X- and Y- momentum equations are :
\[
\frac{\partial U}{\partial Y} = -\frac{K}{\mu} \frac{\partial^2 P}{\partial X \partial Y} + Ra \frac{\partial T}{\partial X}
\]  

The energy equation for porous cavity is :
\[
U \cdot \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}
\]  

Energy equation at the wall:
\[
\frac{\partial^2 T_w}{\partial X^2} + \frac{\partial^2 T_w}{\partial Y^2} = 0
\]  

The heat transfer at the walls are defined as in the following:
\[
Nu = \frac{1}{\frac{A}{H} \frac{\partial T}{\partial X}} \frac{\partial Y}{\partial X}
\]  

The non-dimensional parameters are:
\[
A = \frac{H}{L}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad D = \frac{d}{L},
\]
\[
U = \frac{u L}{\alpha}, \quad V = \frac{v L}{\alpha}, \quad P = \frac{p L^2}{\rho \alpha^2}, \quad T = \frac{T - T_c}{T_h - T_c}
\]
\[
Ra = \frac{g \beta L \Delta T K L}{\nu \alpha}, \quad k_f = \frac{k_{eff}}{k_f} = \varepsilon k_f + \left(1 - \varepsilon\right) k_s
\]

Equations (1)-(6) are solved using non-dimensional initial boundary conditions:

\[
\begin{align*}
& \text{at } X = D \quad U = V = 0 \quad \frac{\partial T}{\partial X} = k_f \frac{\partial T}{\partial X} \\
& \text{at } X = 1 \quad U = V = 0 \quad T_c = 0 \\
& \text{at } X = 0 \quad T_h = 1
\end{align*}
\]

\[
\begin{align*}
& \text{at } Y = 0, A \quad U = V = 0 \quad \frac{\partial T}{\partial Y} = 0
\end{align*}
\]  

2.2 COMPUTATIONAL MODEL

In order to solve the problem with high accuracy and low computation time, different mesh had been tested for the minimum number of elements that leads to grid-independent solution. Fig. 2 illustrates the relation between the average Nusselt number and the resulting number of elements for square porous enclosure for all cases these tested at [Ra=1000, Da=10^3, D=0.1, Kr=1]. It is shown that there is no effect on the average Nusselt number when the number of elements about (80000).

Fig. 1 Schematic diagram of the present work

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The program had been validated regarding average Nusselt number with significant researcher as shown in Table 1. Moreover, for streamlines and isotherms contours, good agreements had been achieved with Saeid (2007) works as shown in Fig. 3.

<table>
<thead>
<tr>
<th>Author</th>
<th>average Nusselt number</th>
<th>Ra = 10</th>
<th>Ra = 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moya, Ramos, and Sen (1987)</td>
<td>1.065</td>
<td>2.801</td>
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<tr>
<td>Beckermann, Viskanta, and Ramadhyani (1986)</td>
<td>no data</td>
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<td>Al-Farhany and Turan (2011)</td>
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<tr>
<td>Ahmed et al. (2016)</td>
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<td>3.01</td>
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<tr>
<td>Present Work</td>
<td>1.08</td>
<td>3.02</td>
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</table>

Table-1: the validation of the present work with significant researchers

3. RESULTS AND DISCUSSIONS

The results of the present work will be displayed considering the effect of the dimensionless parameters like modified Rayleigh number, finite wall thickness, thermal conductivity ratio and the aspect ratio. The results will be presented in terms of streamlines, isotherms, local and average Nusselt number for square enclosure filled with saturated porous medium.

3.1 MODIFIED RAYLEIGH NUMBER

Fig. 4 Demonstrates the isotherms (left) and streamlines (right) for various modified Rayleigh number and \( D = 0.1, K_r = 1, \) and \( E = 0.5 \). It can be noted that as the dimensionless wall thickness increases, the maximum stream function will decreases. For example, when the dimensionless wall thickness increases from \( D = 0.02 \) to \( D = 0.5 \), \( \psi_{max} \) decreases from \( 6.3921 \) to \( 1.2478 \) at \( Ra = 1000 \). The reason is due to the increase in the heat transfer rate when modified Rayleigh number increases. With respect to isotherms, it can be seen that when modified Rayleigh number \( Ra = 10 \), the isotherms have a uniform shape. This is due to the weak effect of convective flow and in this case the conductive heat transfer is dominant. But, when modified Rayleigh number increases to \( Ra = 1000 \), the isotherms change their shapes obviously due to the strong effect of convection heat transfer. Fig. 5 illustrates the profile of local Nusselt number for both the fluid (a) and the solid phase (b) for various values of modified Rayleigh number. As expected, the heat transfer rate enhanced significantly when \( Ra \) increases from 10 to \( Ra = 1000 \) due to the increasing of buoyancy and natural convection flow within the enclosure.

3.2 DIMENSIONLESS WALL THICKNESS EFFECT

Fig. 6 illustrates the isotherms (left) and streamlines (right) for various dimensionless wall thickness and \( [Ra = 1000, K_r = 1] \). It may be noted that as the dimensionless wall thickness increases, the maximum stream function will decreases. For example, when the dimensionless wall thickness increases from \( D = 0.02 \) to \( D = 0.5 \), \( \psi_{max} \) decreases from \( 6.4 \) to \( 6.3921 \). Also, the Nusselt number will decrease because the conduction heat transfer is dominant with increasing wall thickness. For example, \( Nu_w = 6.3921, Nu_{sw} = 12.973 \) at \( D = 0.02 \) while \( Nu_{sw} = 1.2478 \), \( Nu_w = 2.153 \) at \( D = 0.5 \). For the isotherms contours, it can be noticed that as the walls thickness increases, the isotherms pattern becomes more uniform which indicating that the effect of the conduction heat transfer mode becomes more significant. Moreover, the isotherms pattern shows that the heat is transferred from the left sidewall at the middle where the hot wall exists towards the cold right sidewall due to
the large temperature gradient and this result matches with the problem boundary conditions.

Fig. 7 displays effect of dimensionless wall thickness on the local Nusselt number for the fluid phase (a) and solid phase (b). It can be noted that conduction mode will be dominant as the wall thickness increasing which leads to reducing the rate of heat transfer on the enclosure.

![Graphs showing effect of dimensionless wall thickness on local Nusselt number](image)

Fig. 5 Profile of local Nusselt number for (a) the fluid phase and (b) along the active hot wall for various modified Rayleigh number

![Graphs showing profile of local Nusselt number](image)

Fig. 6 Isotherm (left) and streamlines (right) for various dimensionless wall thickness, Ra = 1000, Kr = 1, E = 0.5

(c) $D = 0.5, \bar{Nu}_f = 1.2478, \bar{Nu}_w = 2.153$

![Graphs showing isotherms and streamlines](image)

Fig. 7 Profile of local Nusselt number for the fluid phase and along the active hot wall for various finite wall thicknesses

3.3 THERMAL CONDUCTIVITY RATIO EFFECT

Fig. 8 displays the isotherms (left) and streamlines (right) for various thermal conductivity ratio and $[Ra = 1000, D = 0.1]$. It is known that the thermal conductivity ratio $K_r$ is defined as the ratio of the thermal conductivity of solid walls to the thermal conductivity of the fluid. Therefore, from this definition it can be obtained that when the thermal conductivity ratio $K_r$ is small, i.e., $K_r = 1$, the thermal conductivity of walls is small, too. So, the thermal resistance is high and as a result the average Nusselt number at solid walls is high, while the average Nusselt number of fluids is low. On the contrary, when the thermal conductivity ratio increases from $K_r = 1$ to $K_r = 10$, the thermal conductivity of solid walls increases, i.e., convection decreases, while it decreases for the fluid, i.e., convection increases. For example, when thermal conductivity ratio increases from $K_r = 1$ to $K_r = 10$, maximum stream function value will increases from $\Psi_{max} = 9.7$ to $\Psi_{max} = 15$ respectively. For this reason, the average Nusselt number at solid walls decreases from $\bar{Nu}_w = 6.3712$ at $K_r = 1$ to $\bar{Nu}_w = 1.7256$ at $K_r = 10$.

On the other hand, average Nusselt number for the fluid phase increases

![Graphs showing effect of thermal conductivity ratio on Nusselt number](image)
from $\bar{Nu}_f = 3.3715$ at $K_r = 1$ to $\bar{Nu}_f = 9.9799$ at $K_r = 10$. This result demonstrates that the heat transfer mechanism inside the enclosure is converted from conduction mode when $K_r$ is small into convection mode when $K_r$ is high. Fig. 9 illustrates the effect of thermal conductivity ratio for the fluid phase (a) and along the solid wall (b). The results indicate that when the thermal conductivity ratio increases, the thermal conductivity of the fluid phase decreases. This leads to enhancing of the natural convection heat transfer and increasing the local Nusselt number for the fluid phase. However, this behavior is completely reversed for the Nusselt number along the solid wall. Actually, the local Nusselt number along the hot wall decreases as the thermal conductivity ratio increases which leads to enhancing in the conduction effect.

3.4 EFFECT OF ASPECT RATIO
The effect of aspect ratio on heat transfer rate is presented in Fig. 10. It can be noted that as the aspect ratio increases, stream function value increases. For example $\psi_{max} = 6.3$ at aspect ratio equals to 0.5 while it increases to $\psi_{max} = 41$ at aspect ratio = 10. Fig. 11 illustrates the effect of aspect ratio on average Nusselt number for various modified Rayleigh numbers. At low Rayleigh numbers (Ra<100) when the aspect ratio increases from 0.5 to 1, the average Nusselt number decreases. After that it increases again till aspect ratio increases to 2. After that increasing aspect ratio will lead to decrease in the heat transfer rate. But, at high modified Rayleigh number, when the aspect ratio increases from 0.5 to 1, the Nusselt number will increases. Beyond that, increasing aspect ratio will lead to continuous reducing in the heat transfer rate.
4. CONCLUSIONS
The present work illustrates numerically the natural convection heat transfer in enclosure filled with porous media using Darcy model. The results can be summarized as follow:

1. When the dimensionless wall thickness increases, the convection mechanism will be converted into conduction mode. This will reduce the Nusselt number leading to reduce the rate of heat transfer.
2. When the thermal conductivity ratio increases, the local Nusselt number for the fluid phase will increase. While a reverse behavior for local Nusselt number along the heated wall.
3. As the Rayleigh number increases, the heat transfer rate will be enhanced, as a result, local Nusselt number for both fluid and solid phase will be increased.
4. Increasing of the aspect ratio makes the flow strength increases and the heat transfer decreases.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>d</td>
<td>dimensional wall thickness, m⁻¹</td>
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<tr>
<td>D</td>
<td>non-dimensional wall thickness</td>
</tr>
<tr>
<td>Da</td>
<td>Darcy number</td>
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<tr>
<td>g</td>
<td>gravitational acceleration, m s⁻²</td>
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<td>H</td>
<td>high of the enclosure, m</td>
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<tr>
<td>k</td>
<td>permeability of the porous medium, m²</td>
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<td>k</td>
<td>thermal conductivity, W m⁻¹ K⁻¹</td>
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<td>k₁</td>
<td>thermal conductivity ratio, k₁ = k₁/k</td>
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<td>L</td>
<td>length of the enclosure, m</td>
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<tr>
<td>Nu</td>
<td>Nusselt number</td>
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<tr>
<td>p</td>
<td>pressure, kg m⁻¹ s⁻²</td>
</tr>
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<td>P</td>
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</tr>
<tr>
<td>Pr</td>
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</tr>
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<td>u</td>
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Greek symbols

<table>
<thead>
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<th>Symbol</th>
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<tr>
<td>α</td>
<td>effective thermal diffusivity, m² s⁻¹</td>
</tr>
<tr>
<td>βₜ</td>
<td>coefficient of thermal expansion, K⁻¹</td>
</tr>
<tr>
<td>ν</td>
<td>kinematic viscosity, m² s⁻¹</td>
</tr>
<tr>
<td>ρ</td>
<td>density, kg m⁻³</td>
</tr>
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Subscripts

c | cold |
eff | effective |
f | fluid |
h | hot |
w | wall |

REFERENCES


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